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Solution of the Higgs Scalar-Tensor Theory Without Higgs Particles for Static Stars

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Within the scalar-tensor theory of gravity with Higgs mechanism without Higgs particles, we prove that the excited Higgs potential (the scalar field) vanishes inside and outside of stellar matter for static, spherically symmetric configurations. The field equation for the metric (the tensorial gravitational field) turns out to be essentially the Einsteinian one.

1. INTRODUCTION

A scalar-tensor theory of gravity was developed by Brans and Dicke (1961) in order to introduce some foundation for the inertial mass as well as the active and passive gravitational mass (i.e., the gravitational 'constant') by a scalar function determined by the distribution of all other particles in the universe; the background of this is Mach's principle and the principle of equivalence.

This introduction of mass by a scalar field can now be regarded as a somehow prophetic approach, because in today's Standard Model of particle physics the masses of the elementary particles are generated via the Higgs mechanism, thus using also a scalar field, the Higgs field. The scalar interaction mediated by the Higgs field was investigated by Dehnen *et al.* (1990, see also Dehnen and Frommert, 1991). They showed that any excited Higgs field[2] mediates an attractive scalar interaction[3] of Yukawa type (i.e., short range) between those particles, which acquire mass by the corresponding symmetry breaking (i.e., the fermions and the massive W and Z gauge bosons).

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[2] The quanta of this excited Higgs field are the hypothetical Higgs particles.

[3] This interaction is similar to gravity because it couples to the masses of the particles.

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The Higgs field of particle physics can also serve as the scalar field in a scalar-tensor theory of gravity, as was first proposed by Zee (1979) and more deeply investigated by Dehnen *et al.* (1992). In this theory, in addition to its role in the Standard Model of making the particles massive, the scalar Higgs field also generates the gravitational constant G. Surprisingly, however, if the Higgs field of the $SU(3) \times SU(2) \times U(1)$ Standard Model of elementary particles is employed to generate G, the Higgs field loses its source, i.e., can no longer be generated by fermions and gauge bosons unless in the very weak gravitational channel.

The reader can find the whole formalism of this theory in Dehnen and Frommert (1993).

2. STATIC, SPHERICALLY SYMMETRIC SOLUTIONS OF THE HIGGS SCALAR-TENSOR THEORY

For the excited Higgs field φ, one obtains the following homogeneous, covariant Klein–Gordon equation (Dehnen and Frommert, 1993)[4]:

$$\xi^{|\mu}{}_{\|\mu} + M^2\xi = 0, \qquad \xi = (1 + \varphi)^2 - 1 \tag{1}$$

where M denotes the mass of the Higgs particles in this theory. The field equation for the metric as the tensorial gravitational field reads

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{1 + \xi}\left[T_{\mu\nu} + \frac{v^2}{4(1 + \xi)}\left(\xi_{|\mu}\xi_{|\nu} - \frac{1}{2}\xi_{|\lambda}\xi^{|\lambda}g_{\mu\nu}\right) + V(\xi)g_{\mu\nu}\right]$$
$$- \frac{1}{1 + \xi}\left[\xi_{|\mu\|\nu} - \xi^{|\lambda}{}_{\|\lambda}g_{\mu\nu}\right] \tag{2}$$

with the Ricci tensor $R_{\mu\nu}$ and the Higgs potential

$$V(\xi) = \frac{3}{32\pi G} M^2\left(1 + \frac{4\pi}{3\alpha}\right)\xi^2 \approx \frac{3M^2}{32\pi G}\xi^2 \qquad (\alpha \simeq 10^{33}) \tag{3}$$

$T_{\mu\nu}$ is the energy-momentum tensor of matter.

We now look for the exact solution of this equation for the spherically symmetric and time-independent case. This means that the excited Higgs field is a function of the radius r only, and the metric has the form

$$g_{\mu\nu} = \begin{bmatrix} e^{\nu(r)} & 0 & 0 & 0 \\ 0 & -e^{\lambda(r)} & 0 & 0 \\ 0 & 0 & -r^2 & 0 \\ 0 & 0 & 0 & -r^2\sin^2\vartheta \end{bmatrix} \tag{4}$$

[4]Throughout this paper we use $\hbar = c = 1$ and the metric signature $(+ - - -)$. The symbol $(\ldots)_{|\mu}$ denotes the partial, $(\ldots)_{\|\mu}$ the covariant derivative with respect to the coordinate x^μ.

Using the Christoffel symbols and the Ricci tensor components following from the metric (4) (see, e.g., Landau and Lifshitz, 1992, §100; Tolman, 1934, §98), the nontrivial field equations for the metric read (primes denote derivatives with respect to the radial coordinate r, and $L = 1/M$ is the Compton wavelength corresponding to the Higgs mass M)

$$R_{00} = -e^{\nu-\lambda}\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} + \frac{\nu'}{r}\right)$$

$$= -\frac{e^{\nu-\lambda}}{1+\xi}\left[4\pi G(\rho + 3p)e^{\lambda} - \frac{\nu'\xi'}{2} + \frac{\xi}{L^2}\left(1 - \frac{3}{4}\xi\right)e^{\lambda}\right] \tag{5}$$

$$R_{11} = \frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\nu'\lambda'}{4} - \frac{\lambda'}{r}$$

$$= \frac{1}{1+\xi}\left[-4\pi G(\rho - p)e^{\lambda} - \xi'' + \frac{\lambda'\xi'}{2} + \frac{\xi}{L^2}\left(1 - \frac{3}{4}\xi\right)e^{\lambda}\right] \tag{6}$$

$$R_{22} = e^{-\lambda} - 1 + \frac{r}{2}(\nu' - \lambda')e^{-\lambda}$$

$$= -\frac{1}{1+\xi}\left[4\pi G(\rho - p)r^2 + r\xi' e^{-\lambda} - \frac{\xi r^2}{L^2}\left(1 - \frac{3}{4}\xi\right)\right] \tag{7}$$

and the scalar field equation (1) takes the form

$$\frac{d^2\xi(r)}{dr^2} + \left\{\frac{2}{r} + \frac{1}{2}\frac{d}{dr}[\nu(r) - \lambda(r)]\right\}\frac{d\xi(r)}{dr} = M^2 e^{\lambda(r)}\xi(r) \tag{8}$$

Because of a continuous and finite matter density, i.e., no singularities such as matter points or infinitely thin massive surfaces, we are looking for an exact solution for $\xi(r)$ of this equation which is finite and continuous together with its first derivative.

We can immediately find the exact solution of equation (8) if the metric is the Minkowskian one (perhaps with some constant coordinate transformation). This should be a good approximation for the limit of large distances from the star ($r >> R$, where R is the radius of the star) in the static case. Equation (8) then gets linearized and becomes the usual Klein–Gordon equation for a static, spherically symmetric field:

$$\frac{d^2\xi(r)}{dr^2} + \frac{2}{r}\frac{d\xi(r)}{dr} - M^2\xi(r) = 0 \tag{9}$$

The bounded solution of this equation is the Yukawa function

$$\xi(r) = \frac{Ae^{-r/L}}{r}, \qquad r >> R \tag{10}$$

with A an arbitrary real constant; this is the asymptotic solution for all finite, spherically symmetric systems for large values of r which are asymptotically embedded in flat Minkowski spacetime. The absolute value of this solution is exponentially *decreasing* as $r \rightarrow \infty$.

On the other hand, the spacetime metric is also asymptotically equivalent to the flat Minkowskian one for the limiting case[5] $r \rightarrow 0$. Therefore, the scalar field near $r = 0$ should be given asymptotically again by a solution of equation (9); in this case, the solution should behave regularly at $r = 0$ to avoid singularities. The regular solution at $r = 0$ of (9) is given by

$$\xi(r) = \frac{B \sinh(r/L)}{r}, \qquad 0 \leq r << R \tag{11}$$

(B is another arbitrary real constant), the absolute value of which has a minimum at $r = 0$ and is *increasing* outward.

In addition, we can discuss the limiting case for small values of r more accurately: For the interior solution near the origin at $r = 0$ it is convenient to rewrite the field equation (8) after multiplication with r:

$$\frac{r}{2}\xi'' + \left[1 + \frac{r}{4}(\nu' - \lambda')\right]\xi' = \frac{r}{2}M^2 e^{\lambda(r)}\xi \tag{12}$$

Obviously, for nonsingular fields, $\xi'(r)$ must vanish at $r = 0$. Taylor-expanding $\xi(r)$ as $\xi(r) = \xi_0 + \xi_1 r + \xi_2 r^2 + \cdots$ yields

$$\xi_1 = 0 \tag{13}$$

$$\xi_2 = \frac{M^2}{6} e^{\lambda_0}\xi_0 \qquad [\lambda_0 = \lambda(r = 0)] \tag{14}$$

which shows that the second derivative ξ'' of the scalar field ξ has the same sign at the origin $r = 0$ as $\xi(r = 0)$.

If $\xi_0 = \xi(r = 0)$ is not zero, i.e., ξ does not vanish identically, its absolute value anyway increases outward from the center, i.e., if ξ_0 is positive, ξ

[5] This follows immediately from the requirement that for our spherically symmetric configuration the fields should be differentiable if one considers an arbitrary straight line through the origin: As our fields, ν, λ, and ξ must be spherically symmetric, they must be even functions of the distance from the origin on this line, and thus have vanishing derivatives at $r = 0$, which makes the connection coefficients vanish. It also follows as the limiting case of a corollary based on Birkhoff's theorem that the metric inside an empty central spherical cavity of radius R_i in a spherically symmetric system is equivalent to the flat Minkowski metric for $R_i \rightarrow 0$. This corollary is treated, e.g., in Weinberg (1972), and is also valid in our scalar-tensor theory.

increases, and if it is negative, it decreases outward. One could expect that the complete *exact solution* of equation (8) would have a maximum for every $A > 0$ in equation (10) or $B > 0$ in (11), because it grows when starting from $r = 0$ and vanishes exponentially as $r \to \infty$. On the other hand, its first derivative would vanish at this extremal point, and then equation (8) would force the same sign on the solution $\xi(r)$ and its second derivative. Because the function $\xi(r)$ is positive, one would obtain a minimum and not a maximum at this point. For $A < 0$ or $B < 0$, we have the analogous situation: one expects at least one minimum and gets only a maximum. Therefore, one cannot get the asymptotically bounded exterior solution (10) from any nontrivial solution which behaves regularly near $r = 0$. Thus the only physically permitted static solution is $\xi(r) \equiv 0$, with the constants $A = 0$ and $B = 0$ for the asymptotic solutions.

3. CONCLUSIONS

We have shown that the only physically permitted solution for a static, spherically symmetric configuration in our theory is the trivial one with respect to the scalar field. Therefore, the gravitational tensor field equation becomes an ordinary Einstein equation, so that all calculations for astronomical objects obtained from Einstein's general relativity remain valid. Of course, this approach is only manifest for the exactly spherically symmetric and static case without pointlike singularities, and it does not cover highly dynamic systems (e.g., cosmological models or black holes). Yet it is a good approximation for a great many "normal" objects such as stars, or perhaps all closed systems, e.g., our solar system; for all these our fundamental result should be valid.

As the physical world is dynamic, however, there remains the possibility of dynamic solutions which asymptotically fit a cosmological background (see, e.g., Frommert *et al.*, n.d.). This may be of interest in the context of the dark matter problem.

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